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CHANGE OF PRESSURE IN WELLS OPENING UP HIGHLY INHOMOGENEOUS POROUS COLLECTORS

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We consider a generalized averaged model of the filtration of a weakly compressible liquid in a highly inhomogeneous medium. We construct an approximate analytical solution of the problem that describes the change in the pressure field around a well in a circular bed. We investigate the effect of the parameters of a highly inhomogeneous bed on the form of the well pressure stabilization curve.

1. Generalized Averaged Model of the Process. Highly inhomogeneous is the term applied to a porous medium – a composite whose components differ in permeability by several orders of magnitude. The weakly permeable component of the medium will be called blocks or a matrix and the highly permeable one – a collector or supercollector or cracks (in a rather wide sense).

A particular case of a strong inhomogeneity is the classical cracked-porous medium. In such a case an additional condition is imposed: the void volume of the system of blocks is many times greater than that of the system of cracks. In the present work this limitation is considered to be insignificant.

The first mathematical models of the process of filtration in highly inhomogeneous media related precisely to the case of a cracked porous bed. In the work by Barenblatt and Zheltov [1] it is assumed that the filtration of the fluid is carried out over the system of cracks, while the matrix blocks uniformly distributed in the bed play the role of sources that give off its store of the fluid to the cracks. The intensity of the internal flow of the fluid from the blocks to the cracks was assumed to be proportional to the difference of pressures in each of the media.

Later Warren and Root [2] obtained an analytical solution to the equations, similar to those given in [1], for infinite and finite beds and investigated the behavior of the pressure in a well within the framework of a given model. They noted that in semilogarithmic coordinates the curves of the recovery and stabilization of pressure had two rectilinear identically inclined portions connected by a transitional curvilinear one.

In [3] the effect of the inhomogeneity of the bed over the stratum on the change in the well bottom pressure was investigated. The process of the filtration of a fluid in a bed consisting of two intercalations with substantially different permeabilities was considered. The flow of the fluid to a well that opens up a highly permeable bed is described by an ordinary piezoconductivity equation with an exchange term proportional to the pressure gradient at the boundary of the intercalations. For the Laplace transform of the well bottom pressure an analytical equation was obtained. It was shown that for the large time instants the curve depicting the behavior of the well bottom pressure had a finite rectilinear portion.

Kazemi [4] suggested that a hydrodynamically connected multilayer bed be considered an idealized cracked-porous bed. Then, highly permeable intercalations represented a system of cracks and served as conductors carrying a fluid to the well, while weakly permeable ones represented a matrix which was a feeding medium. On the basis of a numerical model, the behavior of the well bottom pressure in unsteady-state filtration regimes was investigated. It was shown that the nonsteady character of the process of mass exchange between the matrix and cracks exerts a substantial effect on the form of the curve depicting the behavior of the well bottom pressure.

To investigate the behavior of the well bottom pressure in a cracked-porous bed, Boyarchuk and Dontsov [5], in addition to the lamellar model, also considered the block model of the bed. The internal flow function was determined from the solution of the boundary-value problem on a separate block. On the basis of approximate

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analytical solutions they showed that the transitional portion of the well bottom pressure curve can be approximated by a straight line with a slope equal to half the slope of the final rectilinear portion. They noted that at identical characteristic dimensions and with other conditions remaining constant, the pressure recovery curve for the block model lay below that for the lamellar one.

In the work by Swaan [6] the expression for the internal flow function was presented in an integral form for both the lamellar and block models. Within the framework of this model, the problem of the inflow of the fluid to the well was solved only in Laplace transforms and only the initial and final rectilinear portions of the well bottom pressure curve were investigated. An approximate analytical solution of the equations presented in [6] was obtained in [7], where the effect of the parameters of the bed on the form of the well bottom pressure curves was investigated.

In [8], using the method of numerical inversion of the Laplace transform, a solution was obtained for equations similar to those given in [6]. The influence of the memory effect of the well and of the boundary effect was investigated.

In [9], to obtain macroscopic equations of filtration in an inhomogeneous medium having a block structure, the method of averaging processes in periodic media was used extended to the case of a strong inhomogeneity (the method of nonuniform averaging). It was based on two hypotheses: smallness of the characteristic scale of inhomogeneity ε (otherwise the construction of the averaged equations loses meaning) and connectivity of the highly conducting component of the medium. A full series of the averaged models was obtained that differ by the relationship between the parameters ε and ω. For media with weak piezoinhomogeneity the Barenblatt-Zheltov model was obtained and with strong piezoinhomogeneity – that of de Swaan. For the intermediate case, a new "kinetic" model was obtained, in which the exchange process was described by a kinetic relation.

In the present work we investigate the case of strong inhomogeneity described by a model with long-term memory of the following form:

$$\frac{\partial P}{\partial t} - \kappa_2^e \Delta P = -\lambda \int_0^1 \frac{\partial P}{\partial \tau'} K_*(t - T')\, d\tau' ,$$
$$\kappa_2^e = \frac{k_2^e}{\beta_2^* (1-\alpha)\mu}, \quad \lambda = \frac{\beta_1^* \alpha}{\beta_2^* (1-\alpha)} . \tag{1.1}$$

Here P is the pressure in the system of cracks; k_2^e, κ_2^e are the effective permeability and piezoconductivity of the system of cracks; μ is the viscosity of the fluid; Δ is the Laplace operator; α is the fraction of the volume of the bed occupied by weakly permeable blocks; β_i^* is the reduced coefficient of compressibility. In what follows we will assume that $\beta_1^* = \beta_2^*$ and, consequently, λ is the ratio between the amounts of the fluid stored in the matrix and in the cracks.

The exchange process between the matrix and the system of cracks is described by the kernel $K_*(t)$ whose form depends on the geometry of the blocks. For a block in the form of a sphere

$$K(t) = 6/\pi^2 \sum_{n=1}^{\infty} (1/n^2) \exp(-4n^2\pi^2\kappa_1/D^2 t) , \quad -K_*(t) \equiv \frac{dK}{dt} , \tag{1.2}$$

where κ_1 is the piezoconductivity of the matrix; D is the diameter of the block.

2. The Problem of Inflow to a Well. Let us consider an infinite circular bed that has a constant thickness h. The well that opens up this bed operates with a constant discharge Q. Prior to the withdrawal of fluid, the pressure in the bed is constant and equal to P^0.

For the reduced pressure:

$$U(r, t) = \frac{2\pi h k_2^e}{\mu Q} (P^0 - P(R, t)) \tag{2.1}$$

the problem is written in the following form:

$$\frac{\partial U}{\partial t} - \kappa_2^e \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) = -\lambda \int_0^t \frac{\partial U}{\partial \tau'} K_*(t-\tau')\,d\tau' , \tag{2.2}$$

$$U(r, 0) = 0 , \tag{2.3}$$

$$r\frac{\partial U}{\partial r}\bigg|_{r=r} = -1 , \quad U\big|_{r\to\infty} = 0 . \tag{2.4}$$

To solve integro-differential Eq. (2.2) we use the Laplace transform method [10].

The Laplace transform of the function $U(r, t)$, i.e., the function $\overline{U}(r, s) = \int_0^\infty U(r, t)e^{-st}dt$, satisfies a Bessel equation of zero order:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{U}}{\partial r}\right) - \chi\overline{U} = 0 , \quad \chi \equiv s/\kappa_2^e\,(1 + \lambda\overline{K}_*(s)) . \tag{2.5}$$

The overbar indicates the Laplace transform of the corresponding function.

With account for conditions (2.4) it is possible to obtain the solution of Eq. (2.5):

$$\overline{U}(r, s) = -\frac{1}{s}\frac{1}{r_w\sqrt{\chi}}\frac{K_0(r\sqrt{\chi})}{K_0(r_w\sqrt{\chi})} \tag{2.6}$$

or, using the approximation of the Macdonald function $K_0(x)$ for small values of the argument ($x < 0.01$):

$$\overline{U}_w(s) \equiv \overline{U}(r_w, s) = -\frac{1}{2}\left(\frac{1}{s}\ln\frac{e^{2c}r_w^2 s}{4\kappa_2^e} + \Delta\overline{U}(s)\right) , \tag{2.7}$$

$$\Delta\overline{U}(s) = \frac{1}{s}\ln(1 + \lambda\overline{K}_*(s)) , \tag{2.8}$$

where $c = 0.5772...$ is the Euler constant.

In the space of original functions, expression (2.7) is written as

$$U_w(t) = \frac{1}{2}\left(\ln\frac{2.25\,\kappa_2^e t}{r_w^2} - \Delta U(t)\right) , \tag{2.9}$$

where $\Delta U(t)$ is the inversion of the function $D\overline{U}(s)$; in the general case of the function $K(t)$ of form (1.2) this inversion is not determined explicitly.

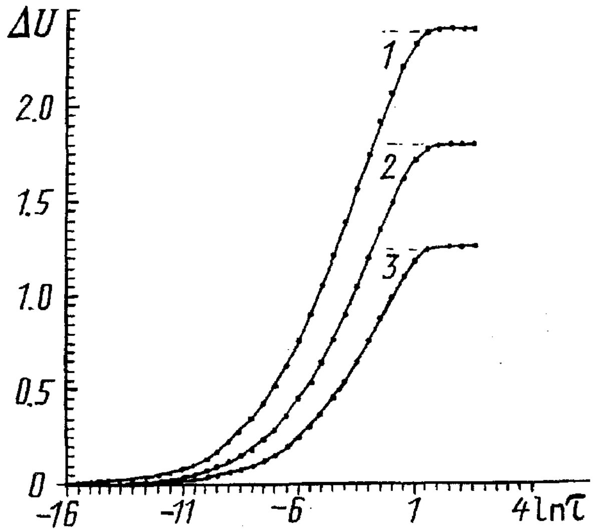
Equation (2.9), which describes the change in pressure at the bottom of a well that opens up a highly inhomogeneous medium, differs from a similar relation for a homogeneous bed by the presence of a function $\Delta U(t)$ that takes into account the internal flow of the fluid from one medium to the other.

For $\Delta U(t)$ we obtain approximate analytical expressions if we use not the function $K(t)$ itself but its approximations.

Let us consider the behavior of the function $\Delta U(t)$ at small times. Letting t in Eq. (1.2) go to zero, we obtain for $t < 0.01t_*$, where $t_* = D^2/(\pi^2\kappa_1)$ is the characteristic time of the development of perturbation in the block, that

$$K(t) \approx 1 - \frac{12}{\pi^{1.5}\sqrt{t_*}}\sqrt{t} . \tag{2.10}$$

After the substitution of Eq. (2.10) into Eq. (2.8) we have



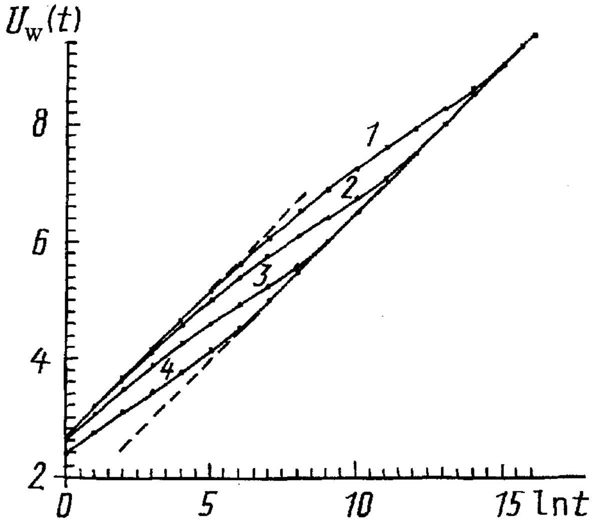


Fig. 1. Dependence of the function $\Delta U(t)$ on ln (τ): 1) $\lambda = 10$; 2) 5; 3) 2.5.

Fig. 2. Change of the reduced pressure in a well in a strongly inhomogeneous bed, $\lambda = 10$: 1) $t_* = 1500$ h; 2) 75; 3) 3.75; 4) 0.188 h.

$$\Delta \bar{U}(s) = \frac{1}{2} \ln (1 + \alpha/s), \quad \alpha = 6\lambda/(\pi/\sqrt{t_*}). \tag{2.11}$$

Since we are interested in small values of t (that correspond to large values of s), in what follows for $\alpha/s < 1$, expanding the logarithm into a series and inverting term by term to the original functions, we find for $\Delta U(t)$, with $t < 0.01 t_*$, that

$$\Delta U(t) = 2/(\sqrt{\pi}) \sqrt{\alpha^2 t} - 0.5\alpha^2 t + 4/(9\sqrt{\pi}) (\alpha^2 t)^{1.5} - \ldots \tag{2.12}$$

At large values of t the series (1.2) converges very rapidly, but starting from $t \approx 0.314 t_*$ all the terms of the series, except for the first one, are negligibly small. Taking this into account, as an approximation of the function $K(t)$ for $t > 0.01t$ we may assume that

$$K(t) \approx 6/\pi^2 \exp(-4t/t_*) + 0.21 \exp(-17.613 t/t_*), \tag{2.13}$$

where the coefficients of the second term are selected so that the standard deviation from the exact function $K(t)$ be minimal over the portion $t/t_* \in [0.01; 1]$.

Substituting Eq. (2.13) into Eq. (2.8), after simple transformations we obtain

$$\Delta U(t) = \ln(1 + \lambda) + \mathrm{Ei}(-\Delta t/t_*) + \mathrm{Ei}(-17.613 t/t_*) -$$

$$- \mathrm{Ei}(-\xi_1 t/t_*) - \mathrm{Ei}(-\xi_2 t/t_*),$$

$$\xi_{1,2} = \frac{1.871 - \lambda/(1 + \lambda) \pm \sqrt{\lambda^2/(1 + \lambda)^2 - 2.014\lambda/(1 + \lambda) + 1.387}}{0.173 - 0.142\lambda/(1 + \lambda)}. \tag{2.14}$$

3. Qualitative Analysis of the Change in Pressure. The graphs of the function $\Delta U(t)$ of the logarithm of the dimensionless time $\tau = t/t_*$ for different values of the parameter λ are given in Fig. 1. In semilogarithmic coordinates the curves have two horizontal asymptotes. When $t \to 0$, the function $\Delta U(t)$ also tends to zero; the time of attainment of the asymptote depends on the parameter a, i.e., depends on both the ratio between the amounts stored in the bed λ and the characteristic time of the development of the perturbation in the block t_*.

The curves attain the second asymptote when $t \sim t_*$, with this time being independent of the parameter λ. In this case, the function $\Delta U(t)$ takes on a value equal to ln $(1+\lambda)$.

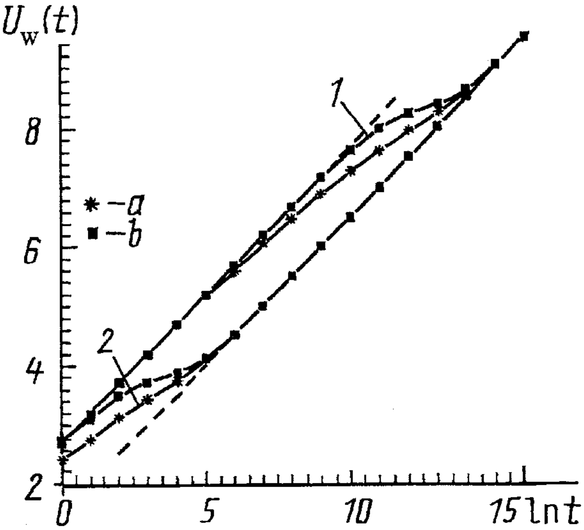


Fig. 3. Effect of the unsteady-state nature of the exchange process on the behavior of the well bottom pressure, λ = 10: 1) t_* = 1500 h; 2) 0.188 h; a) unsteady mass exchange; b) quasistationary mass exchange.

Thus, in the general case, the curve of the stabilization of the well bottom pressure in semilogarithmic coordinates has two rectilinear identically inclined portions connected by an intermediate curvilinear one. The first rectilinear portion corresponds to purely interstitial filtration, i.e., the blocks are not yet involved. This portion is characterized by the effective permeability of the system of cracks k_2^e and by the piezoconductivity of the system of cracks κ_2^e. The second rectilinear portion corresponds to an equivalent homogeneous medium with the effective permeability of the system of cracks k_2^e and reduced piezoconductivity $\kappa_* = \kappa_2^e/(1 + \lambda)$.

Figure 2 presents graphs of the dependence of the dimensionless pressure $U_w(t)$ on the logarithm of t for different values of the characteristic time of development of perturbation in the block t_*, which changes from 62.5 days to 11.3 min. In calculations, the effective piezoconductivity κ_2^e and the well radius r_w were assumed equal to 1 m^2/sec and 0.1 m, respectively.

As noted earlier, curves 1 and 2 display three characteristic portions: two rectilinear and one curvilinear that connects the first two. The vertical deviation of these rectilinear portions from each other is equal to 0.5ln $(1+\lambda)$. However, if the characteristic time of the development of perturbation in the block is small or the relative amounts of the fluid stored in the matrix are large, then only a curvilinear portion can be obtained on the graph of the change in the well bottom pressure. With an increase of time, it approaches asymptotically the second rectilinear portion (curves 3, 4). This is explained by the fact that in the situation considered the role of the matrix is considerable already in the first seconds of the process of decrease in pressure.

We will assume that the pressure behavior curve coincides with the straight line corresponding to purely interstitial filtration if $\Delta U(t) \leq 0.05$. Then, in order that the point of the curve at t = 1 sec lie on the first asymptote, it is necessary that the following condition be fulfilled:

$$t_* \geq 0.13\lambda^2 , \tag{2.15}$$

where t_* is measured in hours, i.e., if λ = 10, the well bottom pressure curve will have the first rectilinear portion only when t_* > 13 h, and if λ = 100, then only when t_* > 1300 h.

It should be noted that the form of the well bottom pressure curve depends substantially on the adopted model of filtration for a cracked-porous bed. Figure 3 demonstrates the difference of the curve depicting the behavior of the dimensionless pressure $U_w(t)$ constructed according to the present model from the curve constructed according to the Barenblatt-Zheltov model that admits the quasistationarity of the exchange process between the matrix and the system of cracks. The latter system has a more compressed transitional curve; at a large time interval it lies much higher. Moreover, for the quasistationary model a much longer first rectilinear portion is typical, whose presence is determined by a relation differing from Eq. (2.15):

$$t_* \geq 0.013\lambda . \tag{2.16}$$

From analysis of Eqs. (2.15) and (2.16) it is seen that in contrast to the quasistationary case, in the case of a transient flow of the fluid in the matrix the well bottom pressure curve has the initial rectilinear portion only at rather high values of the characteristic time of the development of perturbations in the block; this time increases substantially with an increase in the parameter λ. At the same time, for the quasistationary model the characteristic time t_* remains small enough in a wide range of change in the amounts of the fluid stored.

Thus, on the basis of the analysis carried out the following conclusions can be drawn:

1) In the general case the curve of the stabilization of the well bottom pressure in semilogarithmic coordinates displays three characteristic portions: initial and final rectilinear having an identical slope and an intermediate curvilinear portion that connects the first two; the first rectilinear portion corresponds to purely interstitial filtration, and the second to equivalent homogeneous filtration

2) For a cracked porous bed for which the characteristic time of the development of perturbations in the block is small (of the order of several hours) and the ratio between the amounts of the fluid stored in the matrix and in the cracks is high ($\gtrsim 100$), the curve of the stabilization of the well bottom pressure does not have an initial rectilinear portion

3) The well bottom pressure curves for a cracked porous bed predicted by the model (1.1) and by the Barenblatt-Zheltov model differ substantially, which results from the assumption of the quasistationary nature of the process of exchange between the matrix and the cracks in the latter

4) Information about the properties of the matrix and cracks separately can be obtained from the first two portions of the curve (prior to the attainment of the finite asymptote), i.e., over that time interval when the exchange process has not developed as yet; therefore the models involving steady-state exchange are inapplicable for the purpose in principle.

NOTATION

c, Euler constant; D, diameter of a weakly permeable block; Ei, symbol of the integral exponential function; h, bed thickness; $K(t)$, $K_*(t)$, kernels of integral operators; K_0, symbol of the Macdonald function; k, permeability; P, pressure; Q, constant discharge of the well; r, radius; s, complex variable; t, time; t_*, time of the relaxation of a weakly permeable block; $U(r, t)$, reduced pressure; $\Delta U(t)$, function taking into account the internal flow of the fluid from one medium to the other; α, fraction of the volume of the bed per block; β^*, reduced coefficient of compressibility; ε, characteristic scale of inhomogeneity; ω, ratio of piezoconductivities of a block and a crack; Δ, Laplace operator; κ, piezoconductivity; λ, ratio of elastic amounts of weakly permeable blocks and systems of cracks; μ, viscosity of the fluid; τ, dimensionless time; τ', integration variable. Indices: 1, weakly permeable blocks; 2, system of cracks; 0, initial value; w, value in the well; e, effective value; overbar, Laplace transform.

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